Skeletonization Based on Neighborhood Sequences

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Skeletonization provides shape features that are extracted from binary image data [1]. It can be used in raster–to–vector conversion, topological analysis, shape simplification, or feature tracking. In the 2D Euclidean space, the skeleton consists of the locus of the centers of all the maximal inscribed disks [2].

Mathematical morphology is a powerful tool for image processing and image analysis [6]. Its operators can extract relevant topological and geometrical information from binary (and grey-scale) images by using structuring elements with various sizes and shapes. The discrete skeletons can be characterized morphologically: the centers of all maximal inscribed disks can be expressed in terms of erosions and dilations.

The morphological skeleton of set $X \subseteq \mathbb{Z}^2$ by structuring element $Y \subseteq \mathbb{Z}^2$ is defined by

$$S(X,Y) = \bigcup_{k=0}^{K} S_k(X,Y) = \bigcup_{k=0}^{K} (X \ominus Y^k) - ((X \ominus Y^{k+1}) \oplus Y) ,$$

where $\ominus$ and $\oplus$ denote respectively the erosion and the dilation, and

$$Y^k = \begin{cases} \{(0,0)\} & \text{if } k = 0 \\ \{(0,0)\} \oplus Y = Y & \text{if } k = 1 \\ Y^{k-1} \oplus Y & \text{otherwise} \end{cases} ,$$

and K is the last step before X is eroded completely:

$$K = \max\{\ k \ \mid \ X \ominus Y^k \neq \emptyset\ \}.$$

The formulation states that $S(X,Y)$ is obtained as the union of the skeletal subsets $S_k(X,Y)$. It can be readily be seen that the set $S_k(X,Y)$ contains all points $p \in X$ such that x is the center of a maximal "disk" included in X. Note that the limitation of the morphological skeleton is that its construction is based on "disks" of the form Y^k. Hence the morphological skeleton does not provide a good approximation to the Euclidean skeleton.

In the digital space $\mathbb{Z}^2$, two types of motions are considered [5]. The cityblock motion (denoted by $\mathbf{1}$) allows horizontal and vertical movements only, while in the case of chessboard motion (denoted by $\mathbf{2}$) one can diagonal movements, as well. The octagonal distances can be obtained by the mixed use of these motions. The sequences of cityblock and chessboard motions are called neighborhood sequences [3]. Some of them generate metrics on the digital space $\mathbb{Z}^2$ [4].

In order to cut the shortage of the morphological skeleton, we propose a new type of skeleton that is based on neighborhood sequences.

Let $A = (A(i))_{i=1}^{\infty}$ be a $2D$ neighborhood sequence (where $A(i) \in \{\mathbf{1}, \mathbf{2}\}$) and let $\mathcal{Y} = (Y(i))_{i=1}^{\infty}$ be the sequence of structuring elements in which $Y(i)$ corresponding to $A(i)$ is defined by

$$Y(i) = \begin{cases} \{(0,0),(-1,0),(1,0),(0,-1),(0,1)\} & \text{if } A(i) = \mathbf{1} \\ \{(0,0),(-1,0),(1,0),(0,-1),(0,1),(-1,-1),(-1,1),(1,-1),(1,1)\} & \text{if } A(i) = \mathbf{2} \end{cases} .$$

The sequence skeleton of set $X \subseteq \mathbb{Z}^2$ by sequence of structuring elements $\mathcal{Y} \subseteq \mathbb{Z}^2$ is defined by

$$\mathcal{S}(X,\mathcal{Y}) = \bigcup_{k=0}^{K} (X \ominus \mathcal{Y}^k) - ((X \ominus \mathcal{Y}^{k+1}) \oplus Y(k+1)) ,$$

where

$$\mathcal{Y}^k = \begin{cases} \{(0,0)\} & \text{if } k = 0 \\ \{(0,0)\} \oplus Y(1) = Y(1) & \text{if } k = 1 \\ \mathcal{Y}^{k-1} \oplus Y(k) & \text{otherwise} \end{cases}$$

and

$$K = \max\{k \mid X \ominus \mathcal{Y}^k \neq \emptyset\}.$$

Note that the sequence skeleton can be defined in arbitrary dimensions. It is easy to see that

$$\mathcal{S}(X, \mathcal{Y}) = S(X, Y)$$

if $\mathcal{Y} = (Y, Y, \ldots)$. Hence the conventional morphological skeleton is a special case of sequence skeletons.

A novel method for quantitative comparison of skeletonization algorithms is also proposed. According to our experiment, sequence skeletons can provide much closer approximations to the Euclidean skeleton than conventional morphological skeletons do.

References

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